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COHERENT PECULIAR VELOCITIES AND PERIODIC RED SHIFTS

Christopher T. Hill,[a] *Paul J. Steinhardt,*[b] *and Michael S. Turner*[c,d]

[a]Department of Theoretical Physics
Fermi National Accelerator Laboratory
Batavia, IL 60510-0500

[b]Department of Physics
University of Pennsylvania
Philadelphia, PA 19104-6396

[c]NASA/Fermilab Astrophysics Center
Fermi National Accelerator Laboratory
Batavia, IL 60510-0500

[d]Departments of Physics and Astronomy & Astrophysics
Enrico Fermi Institute
The University of Chicago
Chicago, IL 60637-1433

Abstract.

A coherent, sinusoidal peculiar velocity field of amplitude $\delta v/c \simeq 3 \times 10^{-3}$ and wavelength $\lambda \simeq 128h^{-1}$ Mpc could explain the apparent periodicity in red shift seen in the recent pencil-beam survey of Broadhurst, Ellis, Koo, and Szalay (1990). Such a peculiar velocity field could arise if the power spectrum of density perturbations has a strong feature at about this wavelength (e.g., a bump). This explanation has additional predictions: the phase, period, and strength of the periodicity should vary in different directions; the strength of the periodicity should decrease at higher red shifts; and there should be more "thin" structures perpendicular to the line of sight than parallel to it.

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The isotropy of the cosmic microwave background radiation (CMBR), $\delta T/T \lesssim few \times 10^{-5}$ on angular scales ranging from $1'$ to $90°$, provides strong evidence that the mass density of the Universe is smooth on the largest scales, i.e., comparable to the Hubble radius. $H_0^{-1} \simeq 3000h^{-1}\,\mathrm{Mpc}$ (see, e.g., Wilkinson, 1987). (Here the present value of the Hubble parameter $H_0 = 100h\,\mathrm{km\,sec^{-1}\,Mpc^{-1}}$.) On scales less than about $10h^{-1}\,\mathrm{Mpc}$ there is ample evidence that the Universe is quite "lumpy"—the existence of galaxies and clusters of galaxies, and the larger-than-unity amplitude of the galaxy-galaxy correlation function (see, e.g., Peebles, 1980). The structure of the Universe on intermediate scales—from about $30h^{-1}\,\mathrm{Mpc}$ to about $1000h^{-1}\,\mathrm{Mpc}$—is far less certain, as no comparably quantitative database exists. However, there are a number of tantalizing observations that suggest that the structure that exists on these scales is quite rich, including large (greater than $30h^{-1}\,\mathrm{Mpc}$) voids in the distribution of bright galaxies (Kirshner et al., 1981; de Lapparent, Geller, and Huchra, 1986), the larger-than-unity amplitude of the cluster-cluster correlation function on scales up to $30h^{-1}\,\mathrm{Mpc}$ (Bahcall and Soneira, 1983; Bahcall, 1988; however see, Sutherland, 1988, and Olivier et al., 1990), a large-scale, large-amplitude peculiar velocity field in our local vicinity (streaming velocity of order $700\,\mathrm{km\,sec^{-1}}$ on a scale of order $50h^{-1}\,\mathrm{Mpc}$; Dressler et al., 1987), a "Great Wall" at a distance of about $100h^{-1}\,\mathrm{Mpc}$ that covers a large fraction of the sky (Geller and Huchra, 1990), and most recently the discovery of an apparent periodicity in the distribution of red shifts in a pencil-beam survey of the North and South Galactic Polar regions by Broadhurst, Ellis, Koo, and Szalay (1990; hereafter BEKS). The BEKS survey extends to a red shift of about $z \sim 0.5$ in either direction, and there is evidence for a periodicity of $128h^{-1}\,\mathrm{Mpc}$ (in comoving length), which persists over 13 periods.

If the BEKS effect is statistically significant and is found in other directions it would naively suggest that galaxies are clustered on shells roughly centered on our position. Needless to say, such a conclusion is hard to accept, and several "slightly" less radical explanations have been suggested, based upon the idea that the observed periodicity is an "illusion" that arises because of spatially coherent oscillations in the Hubble parameter (Morikawa, 1990a,b; Hill and Turner, 1990; Hill, Steinhardt, and Turner, 1990; Dolgov, 1990) or in one or more of the fundamental constants (Hill, Steinhardt, and Turner, 1990). (Because we look back in time when we look out in space, spatially coherent temporal oscillations give rise to apparent spatial periodicity.) Here we propose an even more modest explanation whose consequences are testable and very different than the "oscillating physics" scenarios. The observed periodicity in red shift can be explained by the existence of a coherent peculiar velocity field of modest amplitude, $\delta v/c \simeq 3 \times 10^{-3}$, and period

$128h^{-1}$ Mpc; we note that such an amplitude is not significantly different from the peculiar velocities inferred for the local neighborhood by Dressler et al. (1987). While such a coherent peculiar velocity field would not arise if the spectrum of density perturbations were gaussian and given by a smooth power law, it could arise if the power spectrum had strong feature at about $128h^{-1}$ Mpc—e.g., a bump. The evidence for structure on comparable scales—voids, walls, supercluster complexes, and coherent flows—suggests that such a bump in the power spectrum may indeed exist. (We also note that in a neutrino-dominated Universe a characteristic scale in the power spectrum arises due to neutrino freestreaming: $\lambda_\nu \simeq 40(m_\nu/30\,\mathrm{eV})\,\mathrm{Mpc}$ (Bond and Szalay, 1983) and leads to a strong cellular structure in the Universe, presumably both in the density and peculiar velocity field (see, e.g., Centrella and Melott, 1982). While the simplest realizations of a neutrino-dominated Universe are most certainly ruled out (see, e.g., White, Frenk, and Davis, 1983a,b), neutrino freestreaming provides at the very least an example of what a sharp feature in the power spectrum can lead to.)

Let us begin by describing how such a coherent velocity field can lead to apparent periodicity in red shifts. Consider the number of galaxies dN_G seen a cone of solid angle $d\Omega$ and red shift interval dz; in the spatially flat Robertson-Walker model

$$\begin{aligned} dN_G &= n_G r(z)^2 dr d\Omega, \\ &= \frac{4H_0^{-3}[1-(1+z)^{-1/2}]^2}{(1+z)^{3/2}} n_G(z) dz d\Omega; \end{aligned} \tag{1}$$

where n_G is the number density of galaxies per comoving volume and $r(z) = 2H_0^{-1}[1-(1+z)^{-1/2}]$ is the comoving radial distance to a galaxy whose red shift is z. For small z, $dN_G \to H_0^{-3}(1-3z+\cdots)n_G(z)z^2 dz d\Omega$. (For cylindrical geometry, the number of galaxies in the comoving cylinder defined by comoving polar radius ρ and red shift interval dz is: $dN_G = \pi\rho^2 H_0^{-1}(1+z)^{-3/2} n_G dz$. For our purposes the precise geometry of the survey is not relevant.) From Eq. (1) it follows that the galaxy number density per comoving volume n_G can be *inferred* from the differential number count $dN_G/z^2 dz d\Omega$:

$$n_G(z) = \frac{dN_G/z^2 dz d\Omega}{4H_0^{-3}[1-(1+z)^{-1/2}]^2/z^2(1+z)^{3/2}}. \tag{2}$$

In the BEKS red shift survey the galaxy number density inferred from the sample appears to exhibit a strong periodicity in the North/South Galactic Polar regions and perhaps in two other directions, although the periods may not be the same (Szalay, 1990).

In deriving Eq. (1) the red shift is *assumed* to be related to the value of the cosmic scale factor at the time the light we see today was emitted from the galaxy in the usual way:

$1+z = R(t)^{-1}$, where R is the cosmic scale factor, which is normalized so that $R \equiv R_0 = 1$ today. If the galaxies in the survey have peculiar velocities, then their measured red shifts z_M are related to their true cosmological red shifts z by

$$z_M = z + v_P/c,$$

where v_P is the component of the peculiar velocity in the direction of the galaxy and for simplicity we will assume that z, $v_P/c \ll 1$. The quantity that allows one to infer galaxy number density, $dN_G/z^2dzd\Omega$, is related to the quantity that is derived from the observations, $dN_G/z_M^2dz_Md\Omega$, by

$$\frac{dN_G}{z^2dzd\Omega} = \frac{z_M^2}{z^2}\frac{dz_M}{dz}\frac{dN_G}{z_M^2dz_Md\Omega}. \tag{3}$$

Since z and z_M differ only by a very small amount, the z_M^2/z^2 factor is not as important as the "Jacobian factor," dz_M/dz, and we shall henceforth ignore the z_M^2/z^2 factor.

Suppose that the peculiar velocity in the direction of interest can be expressed as a sinusoid,

$$v_P = \epsilon \sin\left(\frac{2\pi H_0^{-1}}{\lambda}z + \psi\right), \tag{4}$$

where ψ is a phase factor and ϵ sets the amplitude of the peculiar velocity field. The crucial Jacobian factor dz_M/dz is given by

$$\frac{dz_M}{dz} = 1 + \mathcal{A}\cos\left(\frac{2\pi H_0^{-1}}{\lambda}z + \psi\right); \tag{5}$$

where the amplitude $\mathcal{A} = (2\pi H_0^{-1}/\lambda)\epsilon$. If we take $\lambda = 128h^{-1}\,\mathrm{Mpc}$, then $\mathcal{A} \simeq 150\epsilon/c$ and $\mathcal{A} \sim 0.5$ for $\epsilon \sim 3\times10^{-3}c$. For simplicity, further suppose that the actual number density of galaxies per comoving volume $n_G(z) = n_0$ is spatially uniform. If one uses $dN_G/z_M^2dz_Md\Omega$ in Eq. (2) to infer the galaxy density, one infers an apparent number density of galaxies:

$$n_{\mathrm{APPARENT}}(z) = \frac{n_0}{dz_M/dz} = \frac{n_0}{1 + \mathcal{A}\cos\left(\frac{2\pi H_0^{-1}}{\lambda}z + \psi\right)}. \tag{6}$$

Because of the factor in the denominator, the ***inferred*** galaxy number density would ***appear*** to be periodic and "spiky," even if the actual number density of galaxies is uniform. For values of $\mathcal{A}$ close to unity the inferred galaxy distribution develops cusps, and for $\mathcal{A} = 1$ a caustic forms. For values of $\mathcal{A}$ greater than unity, the single caustic splits into two.

That a very large effect follows from a relatively small amplitude peculiar velocity traces to the fact that the peculiar velocity difference over the scale λ (which is order

ϵ) is comparable to the difference in expansion velocities over the same scale (which is order $H_0\lambda$). To see this, consider a point in space where the peculiar velocity vanishes, but is decreasing with distance from the observer—the kind of node that occurs for a sine function where the argument is equal to π. Galaxies beyond this point will have larger Hubble velocities and negative peculiar velocities—which tend to cancel. Galaxies that are closer have smaller Hubble velocities but larger peculiar velocities—which too tend to cancel. Thus, the red shifts of galaxies closer or farther from the node in v_P tend to the same value, producing an apparent peak in the red shift distribution. The entire effect—especially the formation of caustics—traces to the Jacobian factor dz_M/dz in the denominator of Eq. (6).

The peak-to-trough variation in the number density of galaxies seen in the BEKS survey is at least a factor of three, suggesting that an amplitude of order $\mathcal{A} \sim 0.5$ or greater is required, *assuming* that the actual number density of galaxies is constant. Taking the actual number density of galaxies per comoving volume to be constant is of course a gross oversimplification since galaxies are known to cluster on scales less than about $10h^{-1}$ Mpc—and perhaps on even larger scales. A more realistic comparison of this theoretical explanation and the observations of BEKS would have to take into account the clustering of galaxies and the selection function of BEKS. The key point of our *Letter* is to emphasize that a relatively small amplitude, coherent peculiar velocity field can lead to an apparent "spikyness" and periodicity in the galaxy count vs. red shift because of the Jacobian factor in Eq. (6).

If, as is most plausible, the peculiar velocity field arose due to gravitational effects, then according to linear theory the peculiar velocity is related to the density inhomogeneity on that scale:

$$\left(\frac{\delta v}{c}\right)_\lambda \sim \frac{\lambda H_0}{2\pi}\left(\frac{\delta\rho}{\rho}\right)_\lambda \qquad \Rightarrow \qquad \left(\frac{\delta\rho}{\rho}\right)_\lambda \sim \mathcal{A}. \tag{7}$$

(In linear theory the Fourier components of the velocity perturbation and density perturbation are related by: $|v_k| = R|\dot{\delta}_k|/k$, where overdot indicates a time derivative.) For an amplitude $\mathcal{A} \sim 0.5$, the density inhomogeneity on the scale λ would still be (barely) in the linear regime. Based upon linear theory, the density enhancement would also lead to an enhancement in the apparent galaxy number density by a simple multiplicative factor, $1+\delta\rho/\rho$, which is in phase with the peculiar velocity effect, but which, by its nature, could not produce cusps. In the nonlinear regime, bound lumps form and the velocity dispersion of the lump leads to the so-called finger-of-God effect, rather than a caustic.

Under what circumstances would such a coherent peculiar field arise? Certainly if the power spectrum of the density perturbations ($P(k) = k^3|\delta_k|^2/2\pi^2$) were given by a smooth function, such as a power law in k, such coherence would be unlikely. However, if the power spectrum had a strong feature, such as a bump, the necessary coherence could occur. Such a bump in the power spectrum as been advocated by some to explain the apparent abundance of structure on scales larger than about $10h^{-1}$ Mpc, and can arise in inflationary models with complicated scalar potentials (Salopek, Bond, and Bardeen, 1989), or as a result of a late-time activity in the Universe, for example, a phase transition (Hill, Fry, and Schramm, 1989; Press, Ryden, and Spergel, 1990) or explosions (Ostriker and Cowie, 1981; Ikeuchi, 1981; Hogan, 1983).

Even if a bump exists in the power spectrum, the contribution of many Fourier components in different directions would be unlikely to produce strong periodicity in all directions, especially if the phases of the different components are random. (Periodicity might be expected to arise in a few directions; see, Dekel and Primack, 1990.) However, if the phases were not random—as would be the case with nongaussian perturbations—or if only a few waves contributed—as would be the case if the perturbations arose due to late-time activity in the Universe—such periodicity would be more likely.

If the velocity caustic explanation is correct, it makes additional predictions beyond the current observations. Supposing that the bump in the power spectrum of density perturbations is of finite width, then the apparent period seen in different directions should vary, with the spread in periods depending upon the sharpness of the feature. Further, because several—or even many—Fourier components contribute to the peculiar velocity field, the periodicity may not be exact or even present in all directions and the phase at our position in different directions is likely to be different.

The strength of the Jacobian effect should vary with red shift. In the linear regime, $\mathcal{A} \ll 1$, ϵ grows as $R^{1/2}$ and the peculiar velocity has comoving period λ. Since the amplitude of the effect is proportional to the ratio of the peculiar velocity to the Hubble velocity, the amplitude should vary as $R(t) = (1+z)^{-1}$—i.e., at high red shifts the strength of the periodicity should diminish. In addition, unlike the oscillating physics scenarios, where the periodicity is in time (Hill et al., 1990), here the periodicity is in comoving length.

While some of the "thin" features of the galaxy distribution probably correspond to "real" structures, if the sharpness of the galaxy red shift distribution seen by BEKS is predominantly an "illusion" produced by velocity caustics, then the occurrence of "thin" features in the distribution of galaxies should be predominantly orthogonal to the line of sight, since the Jacobian effect does not arise for components of the peculiar velocity perpendicular to the line of sight. Finally, the presence of power on such a large scale

($\sim 130h^{-1}$ Mpc) may lead to a measurable anisotropy in the CMBR on the angular scale corresponding to the wavelength $\lambda \sim 130h^{-1}$ Mpc—about 1°—provided that the origin of the perturbations is not recent (i.e., after recombination) or washed out by re-ionization of the Universe.

To conclude, our main point is that a relatively modest peculiar velocity field—$v_P \sim 3 \times 10^{-3}c$—could, if it is coherent, have a very striking effect on the *apparent* distribution of red shifts seen in a pencil-beam survey such as that of BEKS. This is because for such an amplitude, the variation in the peculiar velocity over a period (about $130h^{-1}$ Mpc) is comparable to the variation in the Hubble velocity, leading to cusps—and even caustics—in the apparent galaxy distribution. Velocity perturbations of smaller and larger amplitude relative to the Hubble velocity on the same scale would be not so striking: smaller amplitude velocity perturbations ($\mathcal{A} \lesssim 1$) would be less "cuspy" and hence less prominent; and larger amplitude velocity perturbations ($\mathcal{A} \gtrsim 1$) would be associated with bound objects and would lead to the familiar fingers-of-God rather than spikes.

Such an amplitude and length scale for the peculiar velocity field is not inconsistent with other observations of structure on intermediate scales. This explanation for the BEKS effect is both more mundane and has different signatures than the scenarios that involve temporal oscillations (Morikawa, 1990a,b; Hill and Turner, 1990; Hill, Steinhardt, and Turner, 1990; Dolgov, 1990). In the oscillating physics scenarios, one would expect the same spatial periodicity and similar phase in all directions (perhaps modulated by the real structure in the Universe); the amplitude of the peak-to-trough variations would remain constant or increase with red shift; and the actual periodicity is in time, not comoving length. In the peculiar velocity explanation, the periodicity might only be seen in a few directions; both the phase and period could be expected to vary significantly; structures produced by the peculiar velocity effect would appear predominantly orthogonal to the line of sight; and the periodicity is in comoving length.

The analysis presented here is clearly oversimplified; we have neglected the known clumpiness of galaxies, the selection function of the BEKS survey, nonlinear effects (our analysis is done to linear order in z, v_P, and H_0^{-1}/λ), and most importantly we have left unclear the specific origin and nature of the underlying inhomogeneity in the density field that leads to the velocity field. Further observations of a similar nature (i.e., pencil beams) in other directions should serve to clarify matters, and depending upon the outcome, a more complete analysis of the effect we are discussing may be justified.

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